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The $\eta'g^*g^{(*)}$ vertex including the η'-meson mass[⋆]

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Abstract. The $\eta'g^*g^{(*)}$ effective vertex function (EVF) is calculated in the QCD hard-scattering approach, taking into account the η'-meson mass. We work in the approximation in which only one non-leading Gegenbauer moment in both the quark-antiquark and gluonic light-cone distribution amplitude for the η'-meson is kept. The EVF with one off-shell gluon is shown to have the form $F_{\eta'g^*g}(q_1^2, 0, m_{\eta'}^2) = m_{\eta'}^2 H(q_1^2)/(q_1^2 - m_{\eta'}^2)$, valid for $|q_1^2| > m_{\eta'}^2$. An interpolating formulae for the EVF in the space-like region of the virtuality q_1^2, which satisfies the QCD-anomaly normalization for on-shell gluons and the perturbative-QCD result for the gluon virtuality $|q_1^2| \gtrsim 2 \text{ GeV}^2$, is also presented.

1 Introduction

The $\eta'g^*g^{(*)}$ effective vertex function (EVF) [or the η' – gluon transition form factor], $F_{\eta'g^*g^*}(q_1^2, q_2^2, m_{\eta'}^2)$, enters in a number of decays such as $J/\psi \to \eta'\gamma$, $\Upsilon \to \eta'X$, $\Upsilon \to \eta'\gamma$, $B \to (\pi, \rho, K, K^*)\eta'$, $B \to \eta'X_s$, and hadronic production processes, such as $N + N(\bar{N}) \to \eta'X$, and hence is of great phenomenological importance.

At low gluon virtualities, $|q_1^2| \sim |q_2^2| \ll m_{\eta'}^2$, the EVF is determined by the QCD anomaly while in inclusive decays of B- or Υ-mesons, the gluon virtualities can be large enough ($|q_i^2| \gtrsim m_{\eta'}^2$) to allow the perturbative-QCD consideration for the EVF. Indeed, the hard part of the η'-meson energy spectrum in the inclusive decay $\Upsilon(1S) \to ggg^*(g^* \to \eta'g) \to \eta'X$ is well reproduced when the perturbative-QCD form for the η' – gluon transition form factor is used in the analysis [1,2,3].

In the perturbative-QCD framework, the $\eta'g^*g^{(*)}$ EVF can be calculated as a convolution of a hard-scattering kernel with the η'-meson wave-function. For the energetic η'-meson, transverse degrees of freedom of the meson constituents can be neglected and its wave-function is well described in terms of the quark-antiquark and gluonic light-cone distribution amplitudes (LCDAs). As the gluonic content of the η'-meson is very important in many process, its effect can not be ignored. In this approach, the $\eta'g^*g^{(*)}$ EVF has been studied by several groups [4, 5,6,7]. As the η'-meson mass is relatively large, it is not

a good approximation to neglect it in certain kinematical regions, in particular, when the gluon virtuality is time-like. A consistent treatment of the η'-meson mass effect in the EVF was undertaken by us recently [8] and the results obtained are presented in this report.

2 Light-cone η'-meson wave-function

The η'-meson contains both the quark-antiquark and gluonic components and, for energetic η'-meson, its wave-function can be presented in the form of the twist decomposition and described by the LCDAs. Here, we restrict ourselves to the leading-twist (twist-two) approximation only. An underlying theoretical basis for such a description can be found in [6,8]. The twist-two LCDAs are usually used in the following approximate forms:

$$\phi_{\eta'}^{(q)}(u, Q^2) = 6u\bar{u}\left[1 + 6(1 - 5u\bar{u})A_2(Q^2)\right],$$
$$\phi_{\eta'}^{(g)}(u, Q^2) = 5u^2\bar{u}^2(u - \bar{u})B_2(Q^2), \qquad (1)$$

where only the second Gegenbauer moments $A_2(Q^2)$ and $B_2(Q^2)$ are kept. As the quark-antiquark and gluonic components are mixed under the scale evolution, the Gegenbauer moments introduced above are superpositions of the non-perturbative parameters $B_2^{(q)}(\mu_0^2)$ and $B_2^{(g)}(\mu_0^2)$ [the Gegenbauer coefficients]:
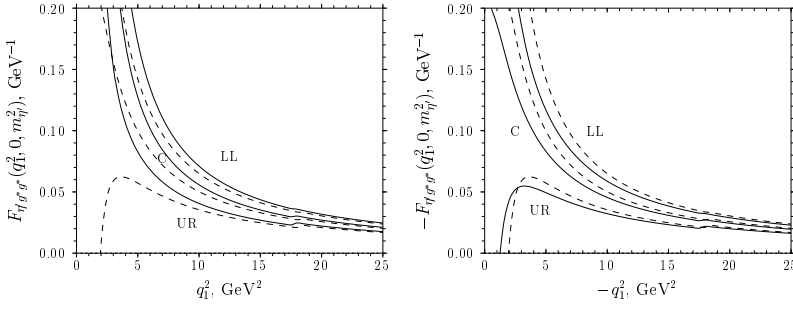
$$A_2(Q^2) = B_2^{(q)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_+^2} + \rho_2^{(g)}B_2^{(g)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_-^2}, \quad (2)$$

$$B_2(Q^2) = \rho_2^{(q)}B_2^{(q)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_+^2} + B_2^{(g)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_-^2}. \quad (3)$$

Fig. 1. The $\eta' g^* g$ EVF for the time-like (left frame) and space-like (right frame) gluon virtuality q_1^2 when the second gluon is on the mass shell ($q_2^2 = 0$). The solid and dashed curves are plotted for the EVF with and without taking into account the η'-meson mass, respectively. The labels C, LL, and UR correspond to the central, lower-left and upper-right points of the combined best fit of the Gegenbauer coefficients presented in Fig. 5 of [3] and given in (4)

The Gegenbauer coefficients at the initial scale μ_0 of the LCDA evolution can be estimated by non-perturbative methods or extracted from the experimental data sensitive to the internal structure of the η'-meson. In particular, a fit to the CLEO and L3 data on the $\eta' - \gamma$ transition form factor for $Q^2 > 2$ GeV^2 was recently undertaken in [6]. The other process which allows to get an independent information on the Gegenbauer coefficients in the $\eta' g^* g$ EVF is the inclusive $\Upsilon(1S) \to \eta' X$ decay. The η'-meson energy spectrum in this decay was recently measured by the CLEO collaboration [1] and the data on the hard part of the spectrum are in agreement with the perturbative-QCD analysis [2,3]. Current experiments and the theoretical analysis undertaken in these processes individually leave an order of magnitude uncertainties on the Gegenbauer coefficients but the combined fit of the data allows to reduce substantially these uncertainties and results in the following values [3]:

$$B_2^{(q)}(2 \text{ GeV}^2) = -0.008 \pm 0.054, \tag{4}$$

$$B_2^{(g)}(2 \text{ GeV}^2) = 4.6 \pm 2.5.$$

3 The $\eta' g^* g^{(*)}$ effective vertex function

In the momentum space, the $\eta' g^* g^*$ EVF can be extracted from the invariant matrix element of the process $\eta'(p) \to g^*(q_1) g^*(q_2)$:

$$\mathcal{M} \equiv F_{\eta' g^* g^*}(q_1^2, q_2^2, m_{\eta'}^2) \, \delta_{AB} \, \varepsilon^{\mu\nu\rho\sigma} \, \varepsilon_{1\mu}^{A*} \varepsilon_{2\nu}^{B*} q_{1\rho} q_{2\sigma}. \tag{5}$$

This amplitude gets contributions from both the quark-antiquark and gluonic components of the η'-meson and the corresponding individual amplitudes $\mathcal{M}^{(q)}$ and $\mathcal{M}^{(g)}$ can be calculated as follows:

$$\mathcal{M}^{(q)} = i f_{\eta'} \int_0^1 du \, \phi_{\eta'}^{(q)}(u, Q^2) \, \mathcal{P}_{j\beta b; i\alpha a}^{(q)} \, \delta^{ab} \big[T_{\rm H}^{(q)} \big]_{ij}^{\alpha\beta}, \tag{6}$$

$$\mathcal{M}^{(g)} = \frac{i f_{\eta'}}{2} \int_0^1 du \, \frac{\phi_{\eta'}^{(g)}(u, Q^2)}{u\bar{u}} \, \mathcal{P}_{\sigma D; \rho C}^{(g)} \, \big[T_{\rm H}^{(g)} \big]_{CD}^{\rho\sigma}. \tag{7}$$

Here, $\big[T_{\rm H}^{(q)} \big]_{ij}^{\alpha\beta}$ ($\big[T_{\rm H}^{(g)} \big]_{CD}^{\rho\sigma}$) is the quark (gluonic) hard-scattering kernel calculated in the perturbative QCD and

$\mathcal{P}_{j\beta b; i\alpha a}^{(q)}$ ($\mathcal{P}_{\mu A; \nu B}^{(g)}$) is the η'-meson projection operator onto the quark-antiquark (two-gluon) state [3].

The results for the quark-antiquark and gluonic parts of the $\eta' g^* g^*$ EVF can be written in the form:

$$F_{\eta' g^* g^*}^{(q)}(q^2, \omega, \eta) = \frac{4\pi\alpha_s(Q^2)}{m_{\eta'}^2 \, \lambda} \, \frac{3 f_{\eta'} \sqrt{N_f}}{N_c} \tag{8}$$
$$\times \left\{ G_0^{(q)}(\omega, \eta) + 6 A_2(Q^2) G_2^{(q)}(\omega, \eta) \right\},$$

$$F_{\eta' g^* g^*}^{(g)}(q^2, \omega, \eta) = -\frac{4\pi\alpha_s(Q^2)}{m_{\eta'}^2 \, \lambda} \, \frac{5 f_{\eta'}}{2\sqrt{N_f}} \, B_2(Q^2) \, \mathrm{G}_2^{(g)}(\omega, \eta), \tag{9}$$

where the following kinematical quantities: $q^2 = q_1^2 + q_2^2$, $\omega = (q_1^2 - q_2^2)/q^2$, $\eta = m_{\eta'}^2/q^2$, and the parameter $\lambda = \sqrt{1 - \eta(2 - \eta)/\omega^2}$ are introduced. The explicit forms of the functions $G_0^{(q)}(\omega, \eta)$, $\mathrm{G}_2^{(q)}(\omega, \eta)$, and $G_2^{(g)}(\omega, \eta)$ can be found in [8]. When the η'-meson mass is neglected ($m_{\eta'} = 0$), the usual $1/q^2$ behavior of the EVF is reproduced [8]. Both contributions contain the factor $1/\lambda$ which is equal to $m_{\eta'}^2/(q_1^2 - m_{\eta'}^2)$ for the case when the second gluon in the final state is on the mass shell ($q_2^2 = 0$). Thus, the phenomenological form for the $\eta' - g$ transition form factor suggested by Kagan and Petrov [9]:

$$F(q_1^2) \equiv F_{\eta' g^* g}(q_1^2, 0, m_{\eta'}^2) = \frac{m_{\eta'}^2 \, H(q_1^2)}{q_1^2 - m_{\eta'}^2}, \tag{10}$$

is naturally reproduced in the LCDA approach when the η'-meson mass is taken into account. As a disadvantage of this approach, one encounters a singularity at $q_1^2 = m_{\eta'}^2$ which, however, can be removed by an inclusion of transverse degrees of freedom for the partons in the η'-meson. In contrast to [9] where it was suggested to approximate the function $H(q_1^2)$ in (10) by a constant value, $H_0 = 1.7$ GeV^{-1}, the explicit form of this function was calculated by us in the framework of the QCD hard-scattering approach [8]. It can be presented the following approximate form:

$$H_{\rm as}(q_1^2) = \frac{4\pi\alpha_s(Q^2)}{m_{\eta'}^2} \sqrt{3} f_{\eta'} \left[1 + A_2(Q^2) - \frac{5}{36} B_2(Q^2) \right], \tag{11}$$

in the region of the applicability of the perturbative QCD which was taken as $q_1^2 < -1$ GeV^2 and $q_1^2 > 2$ GeV^2.
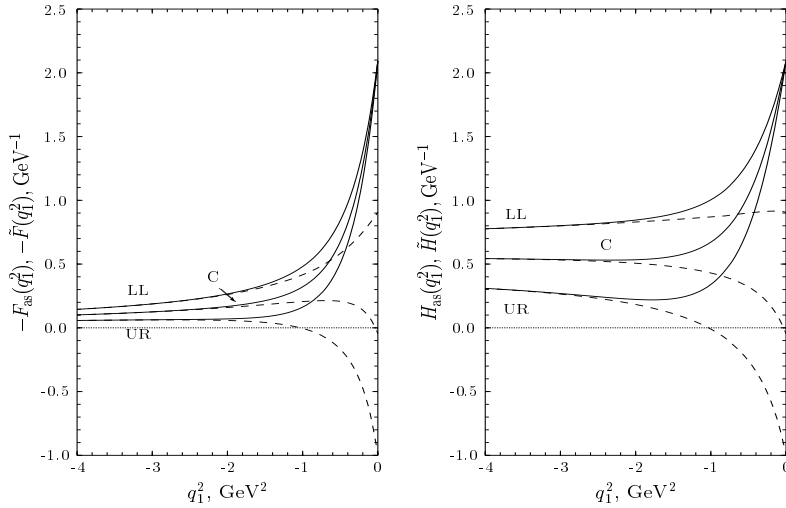
Fig. 2. The $\eta' - g$ transition form factor in the perturbative-QCD approach and using the interpolating formulae for the space-like region of the gluon virtuality q_1^2. The left frame shows the functions $F_{as}(q_1^2)$ (dashed curves) and $\tilde{F}(q_1^2)$ (solid curves). The right frame shows the functions $H_{as}(q_1^2)$ (dashed curves) and $\tilde{H}(q_1^2)$ (solid curves) which are connected with $F_{as}(q_1^2)$ and $\tilde{F}(q_1^2)$ by (10). The labels on the curves are the same as in Fig. 1

The dependence on q_1^2 in the r.h.s. of (11) is coming only through the combination $Q^2 = |q_1^2| + m_{\eta'}^2$.

The dependence of the EVF on the gluon virtuality q_1^2 in the time- and space-like regions for the combined best-fit values (4) of the Gegenbauer coefficients is presented in Fig. 1. The inclusion of the η'-meson mass reduces the parametric dependence on the Gegenbauer coefficients of the $\eta'g^*g$ EVF in the time-like region of the gluon virtuality. This is generally not the case for the space-like gluon virtuality, in particular for the low values, as the η'-meson mass effects are not as pronounced.

A *formal* limit of the function (11) for on-shell gluons ($q_1^2 = 0$) exists with a strong dependence on the Gegenbauer coefficients as shown in Fig. 2. It is well known that the value of the $\eta'g^*g^*$ EVF in this limit is determined by the anomaly:

$$F_{\eta'gg}^{A} = -4\pi\alpha_s(m_{\eta'}^2)\frac{1}{2\pi^2 f_{\eta'}} = -H_A, \qquad (12)$$

which is substantially different from the limiting values of the perturbative-QCD motivated EVF (see Fig. 2). Thus, the behavior of $H_{as}(q_1^2)$ should be modified accordingly. In [8], we suggested the following interpolating formulae:

$$\tilde{H}(q_1^2) = H_{as}(q_1^2) + [H_A - H_{as}(0)]\exp\left[C_s\,q_1^2/m_{\eta'}^2\right], \quad (13)$$

to improve the EVF behavior at the low space-like virtuality. In the numerical analysis, $C_s = 2$ was adopted for the free parameter introduced, which allows a smooth interpolation between the anomaly normalization at $q_1^2 = 0$ and the perturbative-QCD result for the large-$|q_1^2|$ region. As seen from Fig. 2, this interpolation strongly decrease the dispersion in the region of the low space-like gluon virtuality.

4 Summary

The $\eta'g^*g^{(*)}$ EVF is calculated in the perturbative-QCD approach using the LCDAs for the η'-meson wave-function with the inclusion of the η'-meson mass. If one of the gluons is on the mass shell, the pole-like behavior of the η' – gluon transition form factor emerges in this approach for both the quark-antiquark and gluonic parts of the EVF. The corresponding function $H(q_1^2)$ is perturbatively calculated. The η'-meson mass effects are analyzed numerically with the result that they are important for lower values of the gluon virtuality, in particular in the time-like region. An interpolating formulae connecting the QCD-anomaly value and the perturbative-QCD behavior of the η' – gluon transition form factor is presented for the space-like region of the gluon virtuality, taking into account the η'-meson mass, which modifies the EVF significantly in the region $|q_1^2| < 1$ GeV^2 and reduces also the theoretical dispersion in this region considerably.

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